BETTER PRIVACY GUARANTEES FOR DECENTRALIZED FEDERATED LEARNING

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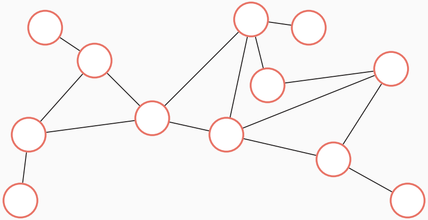
Joint work with **Edwige Cyffers** (Inria Lille), Mathieu Even and Laurent Massoulié (Inria Paris)

Workshop FL-Day - Decentralized Federated Learning: Approaches and Challenges
January 10, 2023

DECENTRALIZED ALGORITHMS: GOOD FOR PRIVACY?



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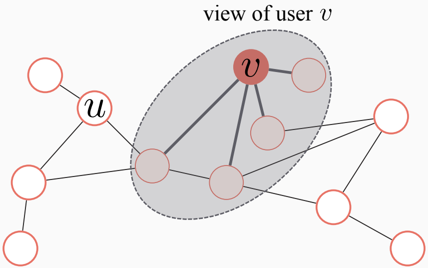

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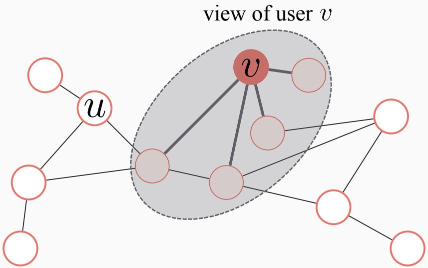

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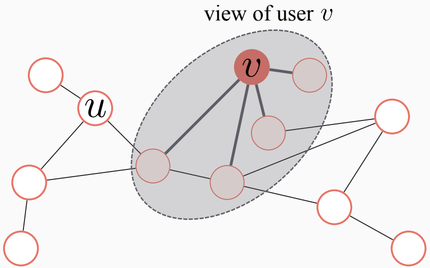

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OUTLINE

1. Background: Differential Privacy & DP-SGD

2. A relaxation of local DP for decentralized algorithms

3. Private random walk-based decentralized SGD

4. Private gossip-based decentralized SGD

5. Conclusion & Perspectives

Background: Differential Privacy & DP-SGD

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PRIVACY ISSUES IN MACHINE LEARNING

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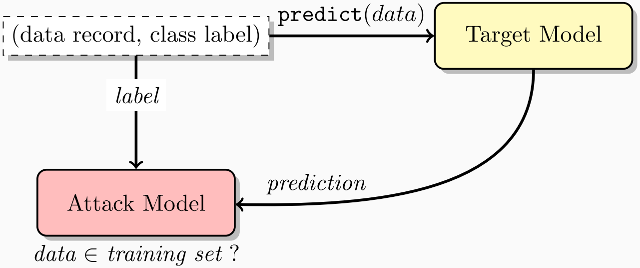
- For instance, one can exploit overconfidence in model predictions [Shokri et al., 2017] [Carlini et al., 2022]



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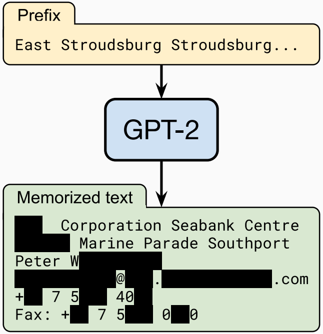
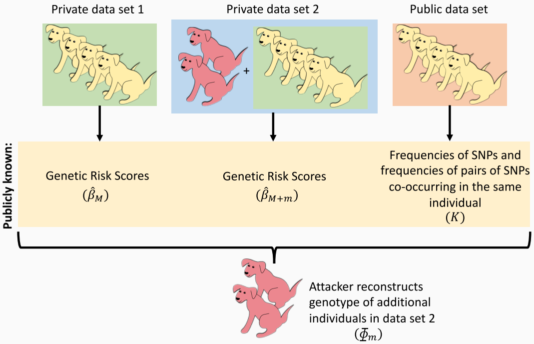
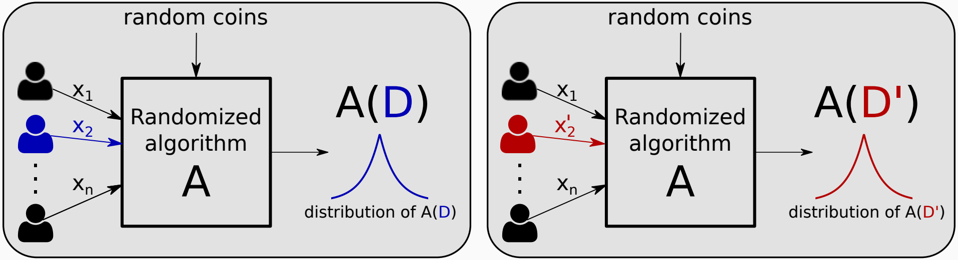
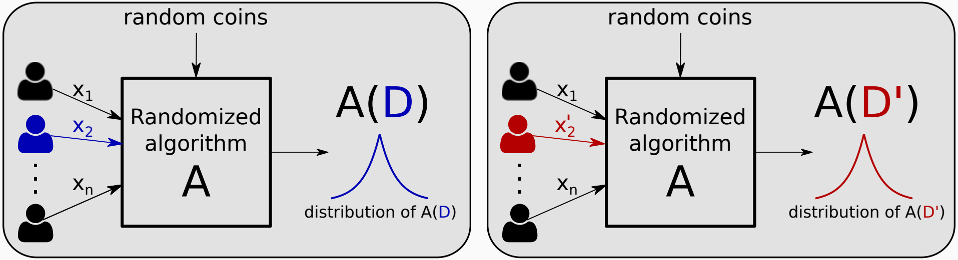
- For instance, one can extract sensitive text from large language models [Carlini et al., 2021] or run differencing attacks on ML models [Paige et al., 2020]







- Neighboring datasets $\mathcal{D} = \{x_1, x_2, \ldots, x_n\}$ and $\mathcal{D}' = \{x_1, x'_2, x_3, \ldots, x_n\}$



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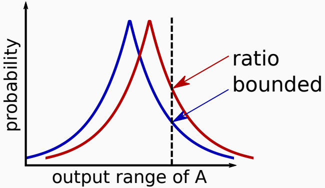
- **Requirement**: $\mathcal{A}(\mathcal{D})$ and $\mathcal{A}(\mathcal{D}')$ should have "similar" distributions



Definition (Rényi Differential Privacy [Mironov, 2017])

An algorithm $\mathcal{A}$ satisfies (α, ϵ)-Rényi Differential Privacy (RDP) for $\alpha > 1$ and $\epsilon > 0$ if for all pairs of neighboring datasets $\mathcal{D} \sim \mathcal{D}'$:

$$D_\alpha\left(\mathcal{A}(\mathcal{D}) || \mathcal{A}(\mathcal{D}')\right) \leq \epsilon\,, \tag{1}$$

where for two r.v. X, Y with densities μ_X, μ_Y, $D_\alpha(X \,||\, Y)$ is the Rényi divergence of order α:

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- Conversion to standard (ϵ, δ)-DP: (α, ϵ)-RDP implies $(\epsilon + \frac{\ln(1/\delta)}{\alpha - 1}, \delta)$-DP for any $\delta \in (0, 1)$

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- Composition: if $\mathcal{A}_1$ is (α, ϵ_1)-RDP and $\mathcal{A}_2$ is (α, ϵ_2)-RDP, then $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ is $(\alpha, \epsilon_1 + \epsilon_2)$-RDP $\rightarrow$ simpler and tighter than composition for (ϵ, δ)-DP

ENFORCING RDP WITH THE GAUSSIAN MECHANISM

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Theorem (Gaussian mechanism)

Let $\sigma > 0$. The algorithm $\mathcal{A}(\cdot) = f(\cdot) + \mathcal{N}(0, \sigma^2\Delta^2)$ satisfies $(\alpha, \frac{\alpha}{2\sigma^2})$-RDP for any $\alpha > 1$.

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Theorem (Subsampled Gaussian mechanism, informal)

If $\mathcal{A}$ is executed on a random fraction q of $\mathcal{D}$, then it satisfies $(\alpha, \frac{q^2\alpha}{2\sigma^2})$-RDP.

- DP induces a privacy-utility trade-off, here in terms of the variance of the estimate
- Random subsampling amplifies privacy guarantees

- A trusted curator wants to privately release a model trained on data $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^n$
- We focus here on approximately solving an Empirical Risk Minimization (ERM) problem under a DP constraint:

$$\min_{\theta \in \mathbb{R}^p} \left\{ F(\theta; \mathcal{D}) := \frac{1}{n} \sum_{i=1}^{n} \ell(\theta; x_i, y_i) \right\}, \quad \text{where loss } \ell \text{ is differentiable in } \theta$$

- Note: in some cases, DP implies generalization [Bassily et al., 2016, Jung et al., 2021]

Algorithm Differentially Private SGD (DP-SGD) [Bassily et al., 2014, Abadi et al., 2016]

Initialize $\theta^{(0)} \in \mathbb{R}^p$ (must be independent of $\mathcal{D}$)
for $t = 0, \ldots, T-1$ **do**
Pick $i_t \in \{1, \ldots, n\}$ uniformly at random
$\eta^{(t)} \leftarrow (\eta_1^{(t)}, \ldots, \eta_p^{(t)}) \in \mathbb{R}^p$ where each $\eta_j^{(t)} \sim \mathcal{N}(0, \sigma^2\Delta^2)$
$\theta^{(t+1)} \leftarrow \theta^{(t)} - \gamma^{(t)}\big(\nabla\ell(\theta^{(t)}; x_{i_t}, y_{i_t}) + \eta^{(t)}\big)$
Return $\theta^{(T)}$

- The sensitivity $\Delta = \sup_\theta \sup_{x,y,x',y'} \|\nabla\ell(\theta^{(t)}; x, y) - \nabla\ell(\theta^{(t)}; x', y')\|$ can be controlled by assuming $\ell(\cdot; x, y)$ Lipschitz for all x, y, or using gradient clipping [Abadi et al., 2016]

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Convex, Lipschitz, smooth loss	$\tilde{O}\left(\frac{\sqrt{p}\ln(1/\delta)}{n\epsilon}\right)$
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- This is optimal [Bassily et al., 2014]: cannot do better without additional assumptions

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- Unfortunately local DP induces a large cost in utility: for averaging n private p-dimensional values in ball of radius Δ under (α, ϵ)-RDP, we have

$$\mathbb{E}[\|x^{\mathrm{out}} - \bar{x}\|^2] = \Theta\Big(\frac{\alpha p \Delta^2}{n\epsilon}\Big) \text{ for local DP}, \quad \text{and} \quad \mathbb{E}[\|x^{\mathrm{out}} - \bar{x}\|^2] = \Theta\Big(\frac{\alpha p \Delta^2}{n^2\epsilon}\Big) \text{ for central DP}$$

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$\rightarrow$ study intermediate models allowing better utility without relying on trusted parties

A relaxation of local DP for decentralized algorithms

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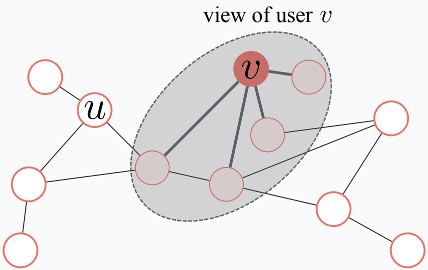

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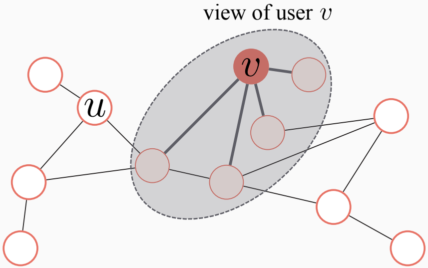

- We want to use this to prove stronger privacy guarantees than under local DP

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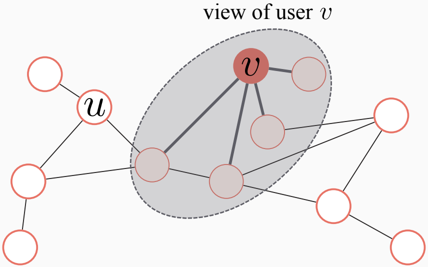
Definition (Network DP [Cyffers and Bellet, 2022])

An algorithm $\mathcal{A}$ satisfies (α, ϵ)-Network DP (NDP) if for all pairs of distinct users $u, v \in \mathcal{V}$ and neighboring datasets $\mathcal{D} \sim_u \mathcal{D}'$:

$$D_\alpha(\mathcal{O}_v(\mathcal{A}(\mathcal{D})) \,||\, \mathcal{O}_v(\mathcal{A}(\mathcal{D}'))) \leq \epsilon\,.$$



- This is a relaxation of local DP: if $\mathcal{O}_v$ contains the full transcript of messages, then network DP boils down to local DP

- We will also consider privacy guarantees that are specific to each pair of nodes, rather than uniform over all pairs

Definition (Pairwise Network DP [Cyffers et al., 2022])

For $f : \mathcal{V} \times \mathcal{V} \to \mathbb{R}^+$, an algorithm $\mathcal{A}$ satisfies (α, f)-Pairwise Network DP (PNDP) if for all pairs of distinct users $u, v \in \mathcal{V}$ and neighboring datasets $\mathcal{D} \sim_u \mathcal{D}'$:

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- For comparison with central and local DP baselines, we will report the mean privacy loss $\bar{\varepsilon}_v = \frac{1}{n} \sum_{u \in \mathcal{V} \setminus \{v\}} f(u, v)$ under the constraint $\bar{\varepsilon} = \max_{v \in \mathcal{V}} \bar{\varepsilon}_v \leq \epsilon$

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- Note: $\bar{\varepsilon}_v$ is not a proper privacy guarantee (we simply use it to summarize our gains)

Private random walk-based decentralized SGD

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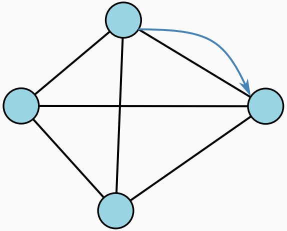

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for $t = 1$ to T **do**
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  $\theta \leftarrow \theta - \gamma[\nabla_\theta F_v(\theta; \mathcal{D}_v) + \eta]$
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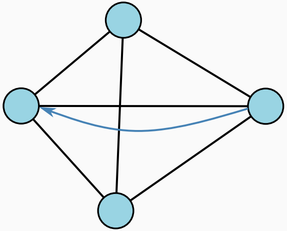

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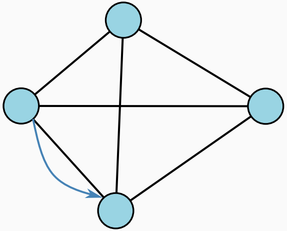

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Theorem ([Cyffers and Bellet, 2022], informal)

Let $F_1(\cdot;\mathcal{D}_1), \ldots, F_n(\cdot;\mathcal{D}_n)$ be convex and smooth. Given $\alpha > 1$, $\epsilon > 0$, let $T = \tilde{\Omega}(n^2)$ and σ^2 be such that private random walk-based decentralized SGD on the complete graph satisfies (α, ϵ)-local RDP. Then the algorithm also satisfies $(\alpha, \frac{\ln^2 n}{n}\epsilon)$-network DP.

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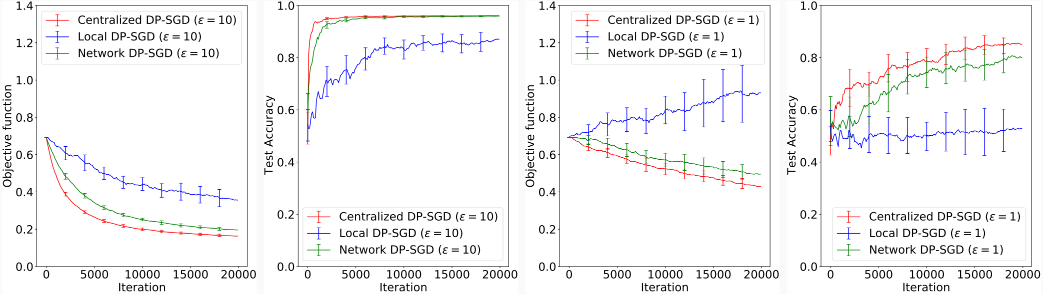
- Utility analysis: same as DP-SGD!
- Privacy analysis: leverages privacy amplification by iteration [Feldman et al., 2018] and exploits the randomness of the walk through “weak convexity” of Rényi divergence

EMPIRICAL ILLUSTRATION



- Numerical results are consistent with our theory: network DP-SGD significantly amplifies privacy guarantees compared to local DP-SGD

Private gossip-based decentralized SGD

GOSSIP-BASED DECENTRALIZED SGD

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- A gossip matrix over the graph $G = (\mathcal{V}, \mathcal{E})$ is a matrix $W \in \mathbb{R}^{n \times n}$ which:
  - is symmetric with nonnegative entries
  - is stochastic, i.e., $W\mathbb{1} = \mathbb{1}$
  - for any $v, w \in \mathcal{V}$, $W_{v,w} > 0$ implies $\{v, w\} \in \mathcal{E}$ or $v = w$

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Algorithm GOSSIP_AVERAGING$(\{x_v\}_{v\in\mathcal{V}}, W, K)$ [Boyd et al., 2006]

for all nodes v in parallel **do**
  $x_v^0 \leftarrow x_v$
for $k = 0$ to $K - 1$ **do**
  for all nodes v in parallel **do**
    $x_v^{k+1} \leftarrow \sum_{w\in\mathcal{N}_v} W_{v,w} x_w^k$, where $\mathcal{N}_v = \{w : W_{v,w} > 0\}$
return $x_1^K, \ldots, x_n^K$

- Consider again $F(\theta; \mathcal{D}) = \frac{1}{n} \sum_{v=1}^{n} F_v(\theta; \mathcal{D}_v)$ with $F_v(\theta; \mathcal{D}_v) = \frac{1}{|\mathcal{D}_v|} \sum_{(x_v, y_v) \in \mathcal{D}_v} \ell(\theta; x_v, y_v)$

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Algorithm Gossip-based decentralized SGD [Lian et al., 2017, Koloskova et al., 2020]

Initialize $\theta_1^{(0)},\dots,\theta_n^{(0)} \in \mathbb{R}^p$
for $t = 0$ to $T-1$ **do**
  for all nodes v in parallel **do**
    $\hat{\theta}_v^t \leftarrow \theta_v^t - \gamma\nabla_\theta\ell(\theta_v^t;x_v^t,y_v^t)$ where $(x_v^t,y_v^t)\sim\mathcal{D}_v$
  $\theta_v^{t+1} \leftarrow \textsc{Gossip\_Averaging}\big(\{\hat{\theta}_v^t\}_{v\in\mathcal{V}}, W, K\big)$
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- Note: to improve the dependence on the topology in the convergence rate we actually use accelerated gossip [Berthier et al., 2020]

- To make the algorithm private, we simply add Gaussian noise before gossiping

Algorithm PRIVATE_GOSSIP_AVERAGING$\left(\{x_v\}_{v\in\mathcal{V}}, W, K, \sigma^2\right)$

for all nodes v in parallel **do**
  $\tilde{x}_v^0 \leftarrow x_v + \eta_v$ where $\eta_v \sim \mathcal{N}(0, \sigma^2)$
$x_1^K, \ldots, x_n^K \leftarrow$ GOSSIP_AVERAGING$\left(\{\tilde{x}_v^0\}_{v\in\mathcal{V}}, W, K\right)$
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Algorithm Private gossip-based decentralized SGD [Cyffers et al., 2022]

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  $\theta_v^{t+1} \leftarrow$ PRIVATE_GOSSIP_AVERAGING$\left(\{\hat{\theta}_v^t\}_{v\in\mathcal{V}}, W, K, \gamma^2\sigma^2\Delta^2\right)$
return $\theta_1^T, \dots, \theta_n^T$

Theorem ([Cyffers et al., 2022])

After K iterations, Private Gossip Averaging is (α, f)-PNDP with

$$f(u,v) = \frac{\alpha\Delta^2}{2\sigma^2}\sum_{k=0}^{K-1}\sum_{w:\{v,w\}\in\mathcal{E}}\frac{(W^k)^2_{u,w}}{\|(W^k)_{w,:}\|^2}$$
$$\leq \frac{\alpha\Delta^2 n}{2\sigma^2}\max_{\{v,w\}\in\mathcal{E}} W^{-2}_{v,w}\sum_{k=1}^{K}\mathbb{P}(X^k = v | X^0 = u)^2,$$

where $(X^k)_k$ is the random walk on graph G, with transitions W.

- As desired, this exhibits the fact that, for two nodes u and v, privacy guarantees improve with their “distance” in the graph

PRIVACY-UTILITY TRADE-OFF OF PRIVATE GOSSIP AVERAGING

- Recall central DP achieves $O(\frac{\alpha p \Delta^2}{n^2 \epsilon})$ and local DP achieves $O(\frac{\alpha p \Delta^2}{n \epsilon})$

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Graph	Arbitrary
Utility (MSE)	$\frac{\alpha p \Delta^2 d}{n^2 \epsilon \sqrt{\lambda_W}}$

- We match the utility of central DP up to an additional $d/\sqrt{\lambda_W}$ factor, where d is the max degree and λ_W of the spectral gap of W

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Graph	Arbitrary	Complete	Ring	Expander
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- Note: we also have extensions to time-varying graphs and randomized gossip

Theorem ([Cyffers et al., 2022])

Let F be μ-strongly convex, F_v be L-smooth and $\mathbb{E}[\|\nabla \ell(\theta^\star; x_v, y_v) - \nabla F(\theta^\star)\|^2] \leq \rho_v^2$. Let $\bar{\rho}^2 = \frac{1}{n}\sum_{v \in \mathcal{V}} \rho_v^2$. For any $\epsilon > 0$, and appropriate choices of T and K, there exists f such that the algorithm is (α, f)-PNDP, with:

$$\forall v \in \mathcal{V}\,, \quad \bar{\varepsilon}_v = \frac{1}{n} \sum_{u \in \mathcal{V} \setminus \{v\}} f(u, v) \leq \epsilon \quad \text{and} \quad \mathbb{E}[F(\bar{\theta}^{1:T}) - F(\theta^\star)] \leq \tilde{\mathcal{O}}\left(\frac{\alpha p \Delta^2 d}{n^2 \mu \epsilon \sqrt{\lambda_W}} + \frac{\bar{\rho}^2}{nL}\right),$$

where d_v is the degree of node v and λ_W is the spectral gap associated with W.

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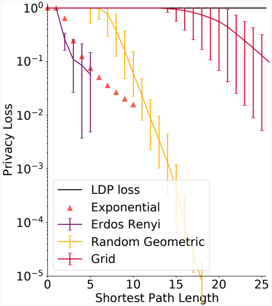
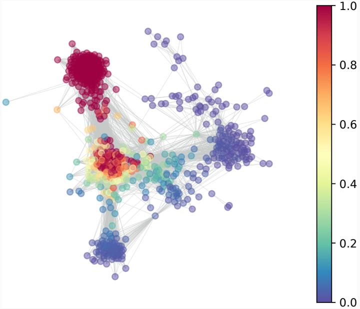
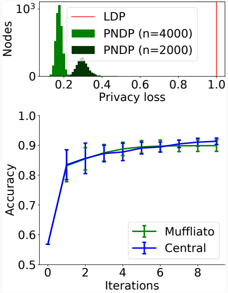
- The first term has the same form as before, so same conclusions apply!
- In particular, with an expander graph, we match the privacy-utility trade-off of centralized SGD with a trusted curator (up to log terms)

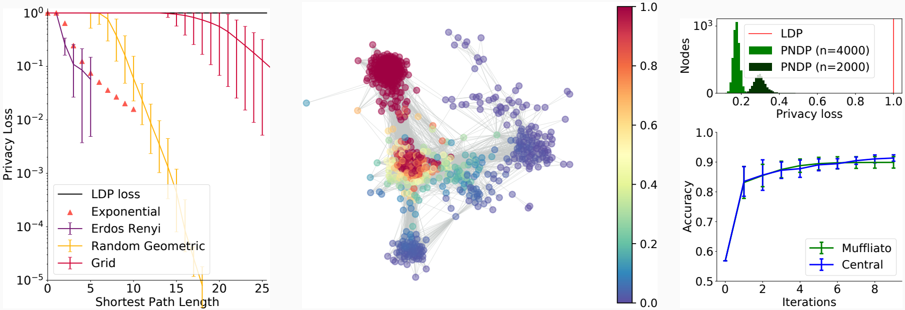






- Users get local DP guarantees w.r.t. their direct neighbors but stronger privacy w.r.t. to other users depending on their distance and the mixing properties of the graph

EMPIRICAL ILLUSTRATION



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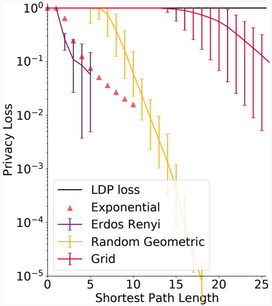

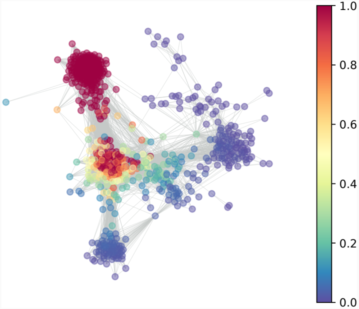

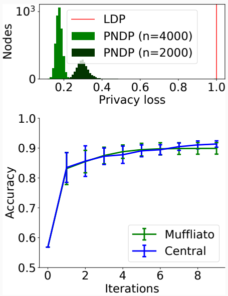

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- This fits the privacy expectations of users in many use-cases (e.g., social networks)
- For learning, we can randomize the graph after each local computation step to make the privacy loss concentrate!

Conclusion & Perspectives

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Perspectives

- Privacy and utility guarantees for random walk-based decentralized SGD on arbitrary graphs [Johansson et al., 2009], possibly with multiple parallel walks [Hendrikx, 2022]

Take-home message

- Decentralized learning can amplify differential privacy guarantees, providing a new incentive for using such approaches beyond the usual motivation of scalability

Perspectives

- Privacy and utility guarantees for random walk-based decentralized SGD on arbitrary graphs [Johansson et al., 2009], possibly with multiple parallel walks [Hendrikx, 2022]
- Capturing the redundancy in gossip-based communication (i.e., correlated noise) to further improve privacy guarantees (recall that even constants matter in DP!)

THANK YOU FOR YOUR ATTENTION!

QUESTIONS?

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